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Strong localization of microwave in photonic fractals with Menger-sponge structure

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Abstract

Photonic fractal with self-similar structure of dielectric medium can strongly localize the electromagnetic wave energy in the three-dimensional fractal structure. We have fabricated Menger-sponge fractal structures of epoxy by using stereolithography. Titania–silica particles were dispersed into the epoxy to increase the dielectric constant and decrease the dielectric loss. The samples have square holes of 1, 3, 9 and 27 mm in edge length through the cubic body of 81 mm. Four types of Menger-sponge samples with fractal stages 1–4 were subjected for measurement of microwave responses. The both reflection and transmission amplitudes were attenuated simultaneously at frequencies of 9.5, 10.5, 12.0 and 13.5 GHz for each sample with fractal stages 1, 2, 3 and 4, respectively. The electromagnetic field amplitudes in the air holes measured by inserting a mono-pole antenna probe confirmed the strong localization of the electromagnetic wave in the central air cavity. The localized mode frequency can be calculated using the sample size, number of stage, and the effective dielectric constant. The localization efficiency increased according to the fractal stage number.

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1. Introduction

Various types of fractal structure with self-similar patterns can be seen in complex forms in nature, such as irregular coastlines, thunderhead, dendritic crystals of snow, etc.[1,2] Their fragments have the similar patterns to the whole structures. The geometric definition of fractal structures was proposed in 1975 by Mandelbrot. Computer simulations using the fractal geometry have been applied to various fields, such as physics, fluid mechanics, image analysis, and even economic prospects. It is pointed out that acoustic wave is localized in a fractal structure.[3] It is very interesting to investigate the wave responses of fractal media.

Electromagnetic responses in fractals exhibit many anomalous behaviors.[3,4] For example, the optical excitations in fractal aggregates of silver colloidal particles localize in small sub-wavelength areas.[5,6] Multiple stop and pass bands for microwaves are excited alternately over a wide frequency range in planar copper-line fractals with sub-wavelength in lateral dimensions as well.[7] Such nonlinear wave excitations are understood mainly due to the self-similar resonances of the secondary radiations produced by surface plasmons, local ohmic currents, or dipolar responses which are induced by the primary incident waves. The dilatation symmetry in the self-similar geometry is thought to bring these wave excitations in small sub-wavelength areas.[6] However, the past experimental studies on electromagnetic responses of fractals were limited to one-dimensional fractal multilayers[8] two-dimensional planars[7] or random fractal objects.[5,6] For the complete confinement of wave excitations, it is necessary to fabricate three-dimensional fractals.

Recently, we have successfully fabricated three-dimensional dielectric fractals called Menger-sponge[2] using a stereolithography CAD/CAM system and found the localization and confinement of microwaves in sub-wavelength regions.[9–11] Titania–silica composite particles were dispersed into epoxy to obtain fractal media with higher dielectric constant. The Menger-sponge structure is known as a three-dimensional version of a Cantor-bar fractal. The fractal dimension is 2.73. The

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stage 3 Menger-sponge fabricated has three different square through holes (1, 3, and 9 mm in edge lengths) normal to each cube face of a dielectric cube with 27 mm × 27 mm × 27 mm in size. Both transmission and reflection amplitudes of incident microwaves normal to the Menger-sponge made of epoxy attenuated largely to about −40 dB at the same frequency of 8 GHz. The intensity profile of electric field measured suggested that the incident microwave at 8.0 GHz is confined in the central air cavity of the Menger-sponge fractal. The wavelength of the localized mode corresponded to the 2/3 of the optical length of the Menger-sponge cube.

Because no such fractal structure was known which can strongly localize and confine electromagnetic waves, we named it as photonic fractal. The purpose of this study is to achieve more strong localization and confinement of microwaves by increasing the fractal stage. Four types of the Menger-sponge samples composed of the titania–silica particles dispersed epoxy with fractal stages 1–4 were fabricated. The microwave attenuations of transmission and reflection amplitudes for these samples and the intensity profiles of the electric field at the interior and exterior air space were measured. The relation between the localized electromagnetic mode and the stage number will be discussed.

2. Experimental procedure

The three-dimensional fractal with Menger-sponge structure was designed using a CAD program (Toyota Keram Ltd., Think Design Ver. 8.0). The design process is illustrated in Fig. 1a–d. A cubic initiator was cut into identical 27 (3 × 3 × 3) smaller cubes on a computer. Seven smaller cubes are extracted from the face and body center positions. This cutting and extracting step is repeated i times to form a stage i Menger-sponge. The fractal dimension is calculated as $D = \log N / \log S$, where N and S are the number of remaining cubes and the divided number of the edge length for one step, respectively. In this study, the stages 1–4 Menger-sponge with the cube edge length of 81 mm and square through holes of 1, 3, 9 and 27 mm in edge lengths were designed.

The real Menger-sponge samples was processed by using a stereolithographic machine (D-MEC Ltd., SCS-300P).[12] The

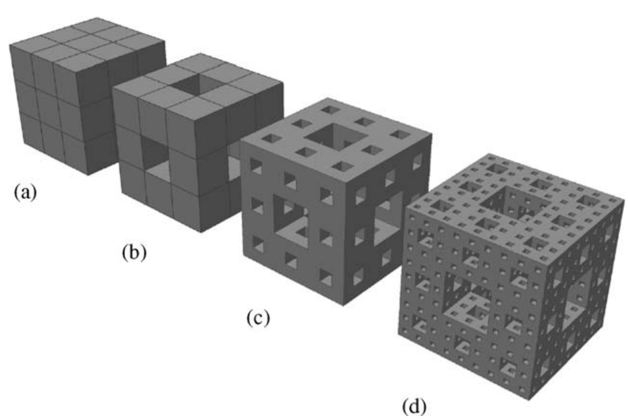

Fig. 1. Schematic illustrations of three-dimensional fractals with a Menger-sponge structure: (a) stage 0; (b) stage 1; (c) stage 2; (d) stage 3.

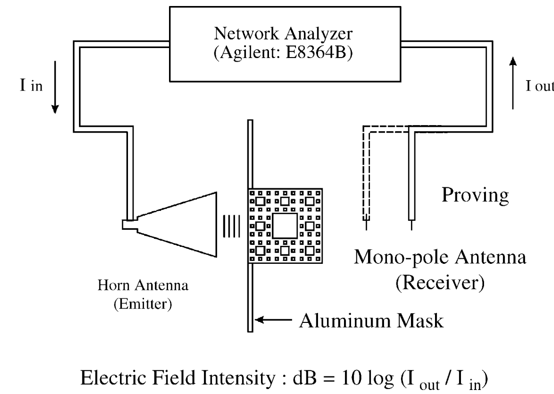

Fig. 2. An experimental configuration to prove the intensity profile of the electric field inside and outside the Menger-sponge structure.

CAD model was converted into STL files and sliced into thin sections. The titania–silica ceramic particles with 10 μm in average diameter were dispersed into the photosensitive liquid epoxy-resin at 10 vol.%. An ultraviolet laser beam of 350 nm wavelength was scanned along the liquid surface to draw the sliced section of the Menger-sponge. The laser spot diameter was 100 μm and the scanning speed was 90 mm/s. Each layer of 150 μm in thickness was solidified through photopolymerization and stacked layer by layer to create a three-dimensional object. The dimensional accuracy of the object compared to the model was within 0.15%. The microstructure of the Menger-sponge was observed by using SEM.

The microwave attenuations of the transmission and reflection amplitudes for the Menger-sponge samples were measured by using two horn antennas and a network analyzer (Agilent Technologies, E8364B).[13] The dielectric constant of the titania–silica/epoxy composite was measured for the bulk sample by using a dielectric probe test kit (Agilent Technologies, HP85070B). The intensity profile of the electric field inside and outside the Menger-sponge of stage 4 was measured by using a prove mono-pole antenna and a network analyzer. The measured intensity in a free space without sample was calibrated as 0 dB. The source horn antenna was fixed at an interval of 150 mm away from the sample as shown in Fig. 2. The microwave was emitted normal to the Menger-sponge through the aluminum window with 81 mm square and 0.5 mm thick.

3. Results and discussion

A stage 4 Menger-sponge object composed of titania–silica particles dispersed into epoxy is shown in Fig. 3. Others Menger-sponge samples with fractal stages 1–3 were precisely fabricated according to the CAD model. SEM observation showed that the ceramic particles were uniformly dispersed in the epoxy. The dielectric constant measured for a bulk sample with the same composition of the Menger-sponge objects was 8.8.

The transmission and reflection spectra of microwaves are shown in Fig. 4. The both amplitudes dropped sharply at different frequencies of 9.5, 10.5, 12.0 and 13.5 GHz depending on the fractal stages 1, 2, 3 and 4, respectively. The attenuation peaks were deeper with increasing the stage number. The max-

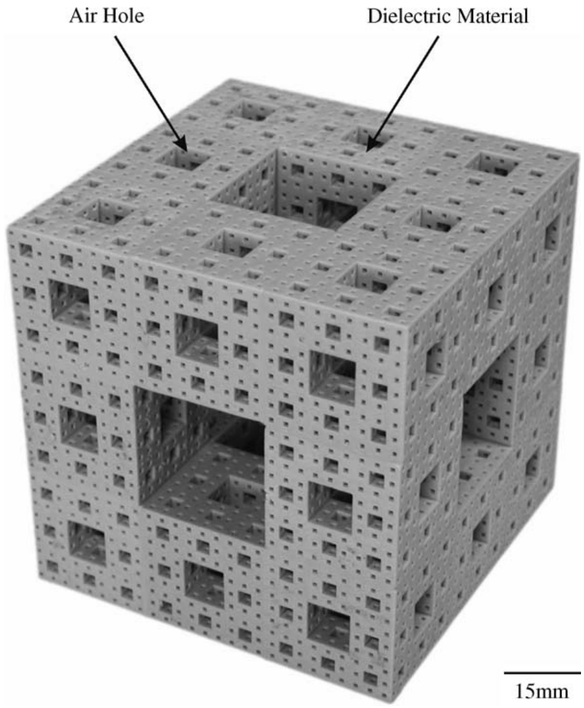

Fig. 3. A photonic fractal with the Menger-sponge structure composed of epoxy incorporating titania–silica dielectric particles.

imum attenuation of both transmission and reflection reached about −50 dB at the stage 4. The bulk sample with the same size and composition as those Menger-sponges showed almost flat profiles without attenuations in transmission and reflection spectra.

In our previous studies on the localization of electromagnetic wave in the dielectric Menger-sponges, an empirical equation was proposed to estimate the wavelength of the localized mode as follows.[11,12]

$$\lambda = \frac{2^{\ell}}{3^{2\ell-1} a\sqrt{\varepsilon_{\text{eff}}}} \tag{1}$$

where a, ε_{eff} and ℓ are the edge length of the Menger-sponge cube, the effective dielectric constant and the order number of the localized mode, respectively. The effective dielectric constant is defined as the mean dielectric constant of the Menger-sponge structure, $\varepsilon_{\text{eff}} = \varepsilon_{\text{m}} f + \varepsilon_{\text{a}}(1-f)$, where ε_{m} and ε_{a} are dielectric constants of the object material and air, respectively. f is the volume fraction of the material of Menger-sponge, which is calculated from the formula of $f = (N/S^m)^n$, where m and n are the real dimension of Menger-sponge objects and the stage number, respectively. In the case of Menger-sponge with fractal stages 1–4, the effective dielectric constants ε_{eff} were calculated to be 6.8, 5.3, 4.2 and 3.4, respectively. The frequency of the second order localized mode ($\ell = 2$) estimated from Eq. (1) were 9.6, 10.8, 12.2 and 13.6 GHz. These values agreed well with the measured frequency as seen in Fig. 4.

The intensity profile of the electric field of the microwave at 13.5 GHz which was measured in the interior and exterior air space of the stage 4 Menger-sponge is shown in Fig. 5. The maximum intensities formed a double shell structure with a truncated cube shape in the central air cavity. In our previous investigation,[11] the Menger-sponge structure with 27 mm in cubic size formed the first order localized mode ($\ell = 1$) with the maximum intensities of a single cube shell at 8 GHz. These result may indicate that the order number of the localized mode ℓ correlate with the electromagnetic mode profile in the central air cavity. In the exterior space around the Menger-sponge structure, no electric field intensity over about −60 dB was detected.

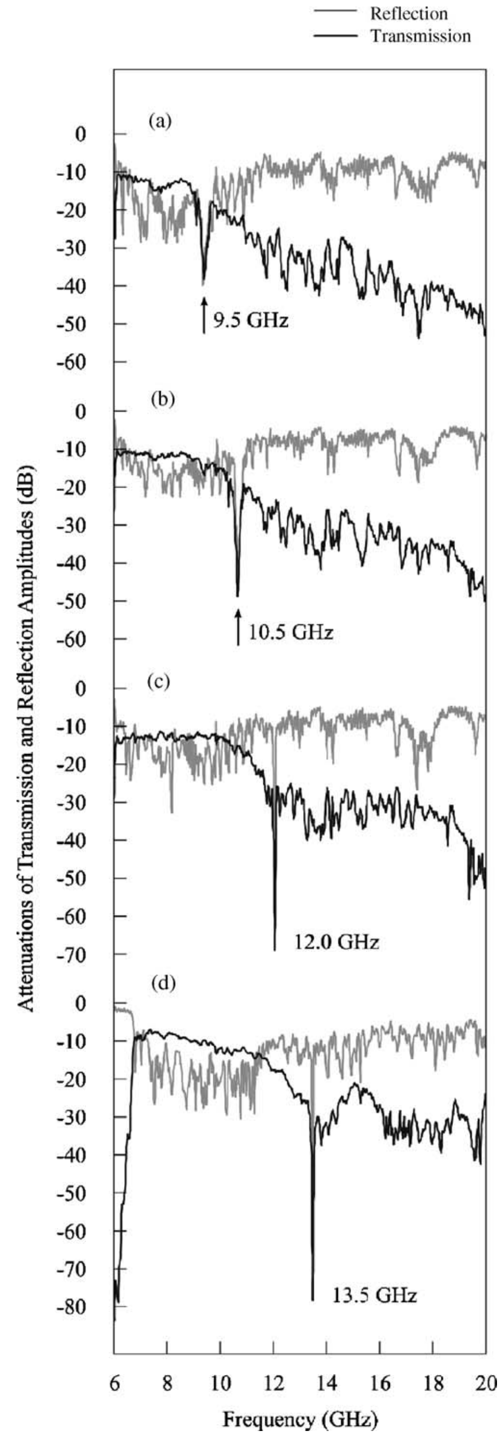

Fig. 4. Transmission and reflection spectra of the photonic fractals with the Menger-sponge structure: (a) stage 1; (b) stage 2; (c) stage 4; (d) stage 4.

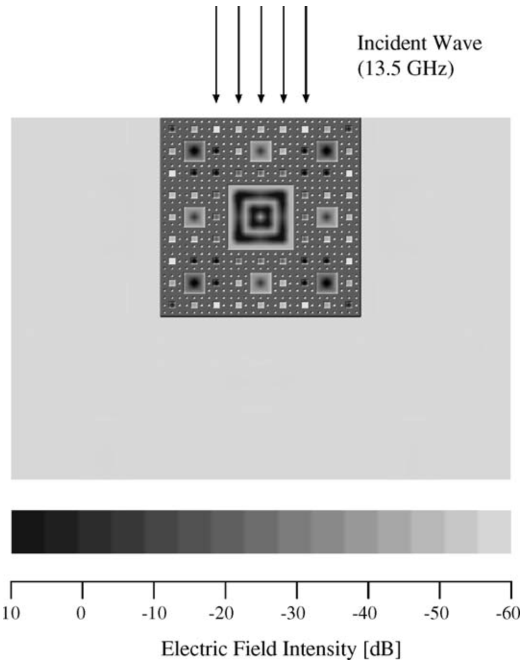

Fig. 5. An intensity profile of the electric field at the localized frequency inside and outside of the Menger-sponge.

It is not understood yet why such a strong localization occurs in the 3D dielectric Menger-sponge fractal. The finite-difference time-domain (FDTD) simulation is carried out to explain these experimental results. The relaxation time of the localized mode is also analyzed experimentally and theoretically. The theoretical study on the localization mechanism in such fractals will be reported elsewhere.

4. Conclusions

Photonic fractals having different Menger-sponge structures of stages 1–4 having the same edge length of 81 mm were designed by CAD and fabricated with epoxy including 10 vol.% titania–silica particles by stereolithography. Each structure showed attenuations of transmission and reflection of microwave at the same frequency, whose amplitude gradually increased with increasing the stage number showing strong localization of microwave in higher stage Menger-sponge structures. The measured frequencies were dependent on the stage number which controls the effective dielectric constant of the Menger sponges. The calculated frequencies of the localized modes in each stage showed good agreements with the measured frequencies. The calculation showed that the localized modes are the second order. The measured intensity profiles of electric field of the localized mode in interior and exterior air space confirmed that the strong localization exists in the central air cube.

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